Critical Curves of a Triple Lens with Fixed Mass Ratios

K. Daněk and D. Heyrovský

Institute of Theoretical Physics, Charles University,
Prague, Czech Republic.

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Outline

Motivation:

- Already observed
- No general analysis of triple lens
- Aid for light-curve interpretation

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Binary lens:

- 2 parameters: mass ratio μ, separation d
- critical-curve topologies: 3
- critical-curve topologies correspond to caustic topologies

Triple lens:

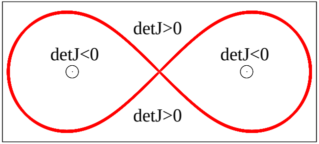
- 5 parameters: 2 mass ratios, 3 lens-position parameters
- critical-curve topologies: ?
- caustic topology analysis: additional cusp counting

Changes in critical-curve topology

Triple-lens topology: 1-5 loops in different configurations

Lensing equation:

$$\zeta = z - \sum_{i=1}^{3} \frac{\mu_i}{\bar{z} - \bar{z}_i}$$



Critical-curve equation:

$$\det J = 1 - \left|\frac{\partial \zeta}{\partial \bar{z}}\right|^2 = 0. \qquad \rightarrow \qquad \sum_{i=1}^{3} \frac{\mu_i}{(z - z_i)^2} = e^{-2i\phi}.$$

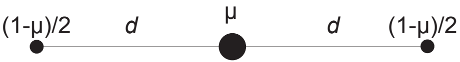
Merging condition: Lens parameters permitting common solution of $\det J = 0$ and saddle-point equation

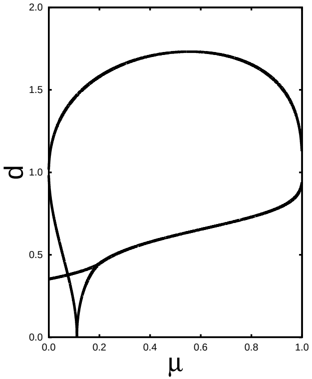
$$\sum_{i=1}^{3} \frac{\mu_i}{(z - z_i)^3} = 0.$$

Analysis of two-parameter triple-lens models

Sylvester matrix method used (Erdl & Schneider 1993)

Example: Collinear symmetric configuration with variable mass





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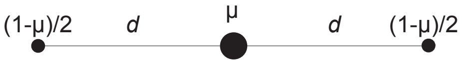

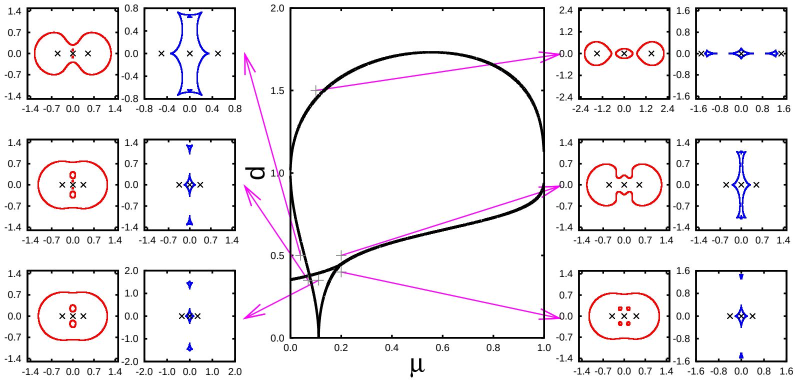

Contour lines and scaling of lens positions

Jacobian contour line $\det J = 1 - S^2$ is given by:

$$\sum_{i=1}^{3} \frac{\mu_i}{(z - z_i)^2} = S e^{-2i\phi}.$$

In "resized" coordinates: $z' = \sqrt{S}\, z$, $z_i' = \sqrt{S}\, z_i$:

$$\sum_{i=1}^{3} \frac{\mu_i}{(z' - z_i')^2} = e^{-2i\phi}.$$

critical curve!

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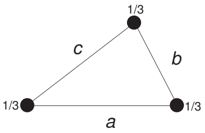
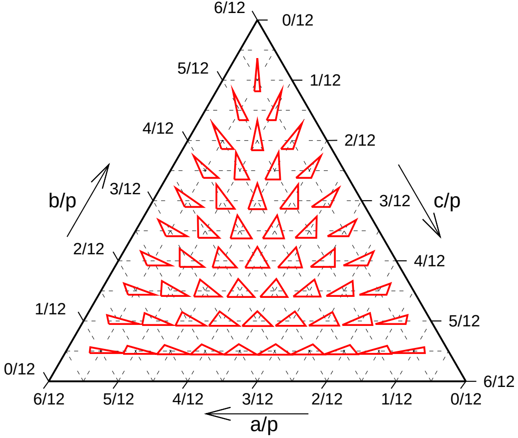
To find conditions for critical-curve loop merger we:

1. Start from some chosen lens parameters.
2. Find six values of $\det J$ in saddle points.
3. Determine six "resized" lens configurations with merging critical-curve loops.

Three-parameter model: Ternary plot

Two shape parameters and perimeter

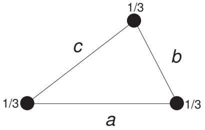


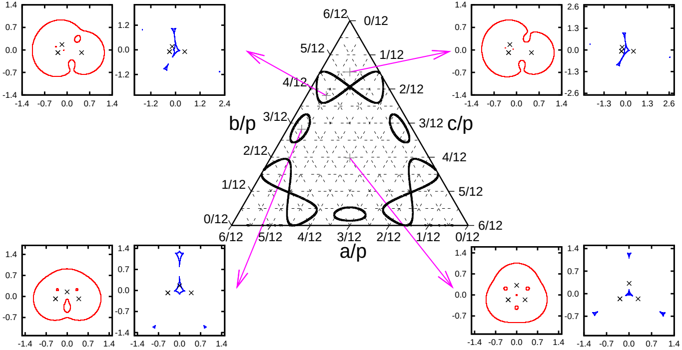


Three-parameter model: Triple lenses with equal masses

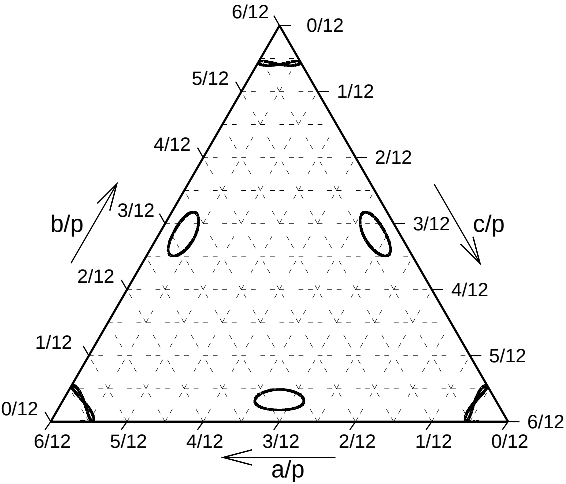
Sequence of ternary plots for different values of perimeter.

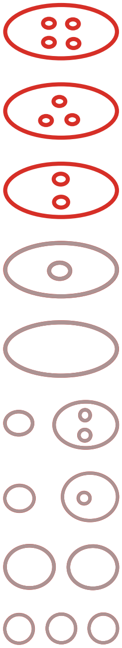
Example: $p = 1.68$



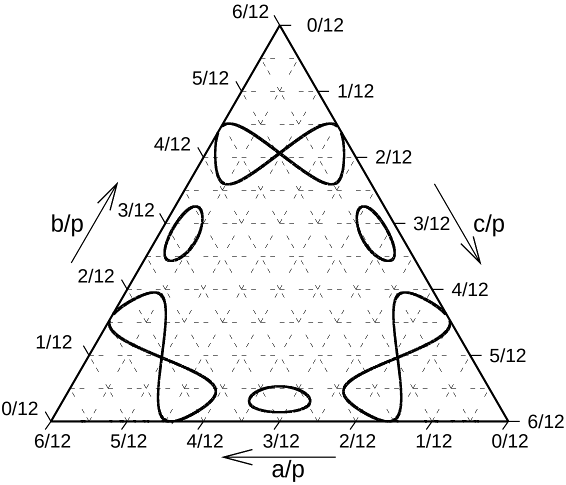


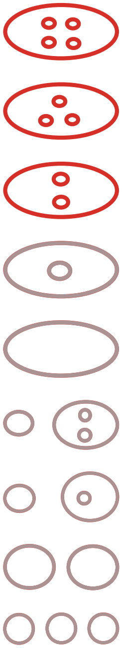
Perimeter cuts gallery: p=1.50



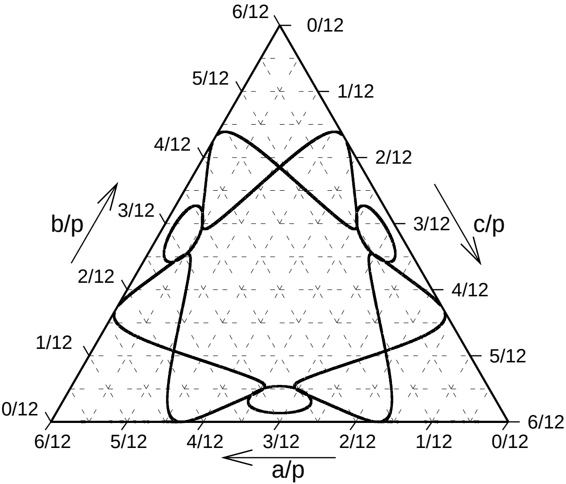


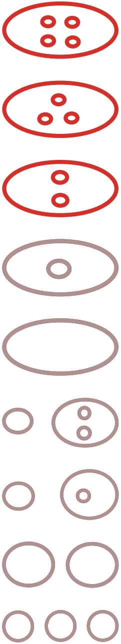
Perimeter cuts gallery: p=1.68



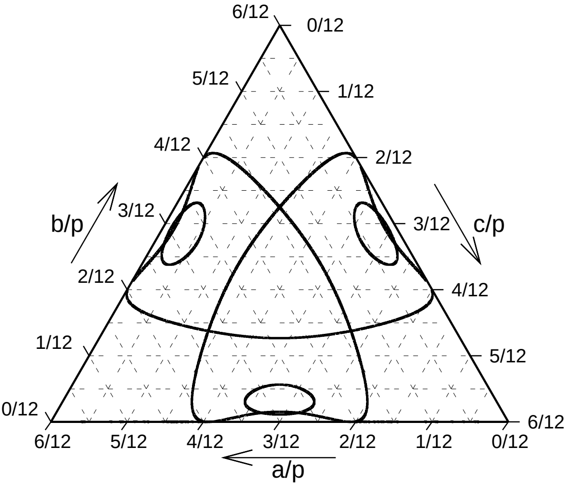


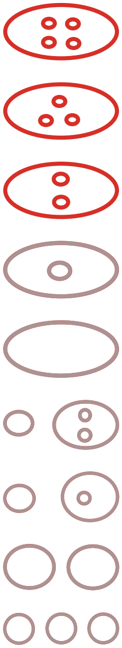
Perimeter cuts gallery: $p=1.71$



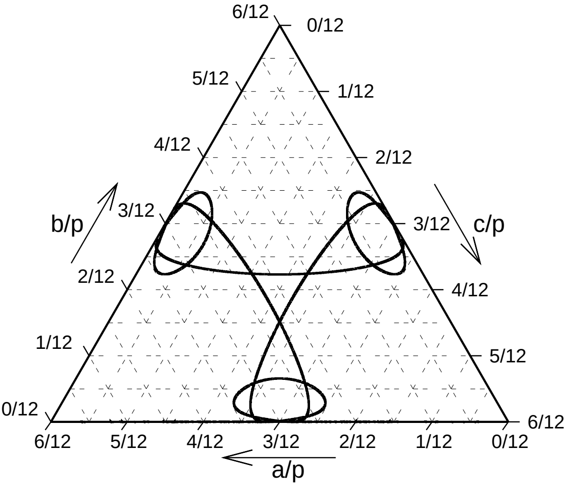


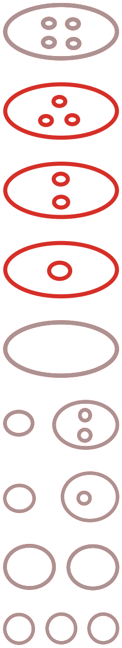
Perimeter cuts gallery: p=1.80



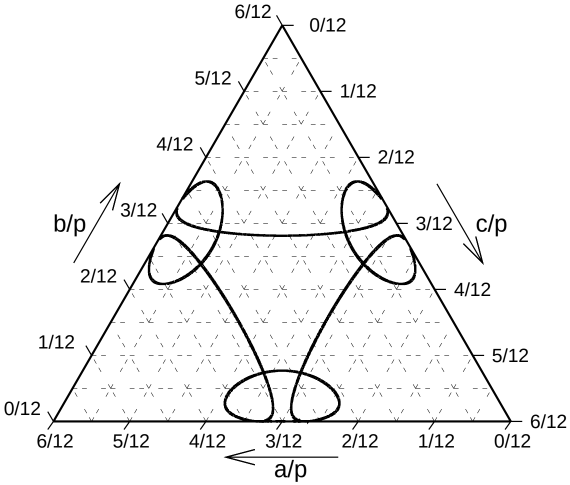


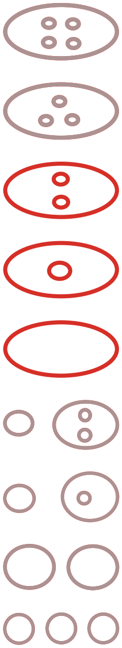
Perimeter cuts gallery: p=2.10



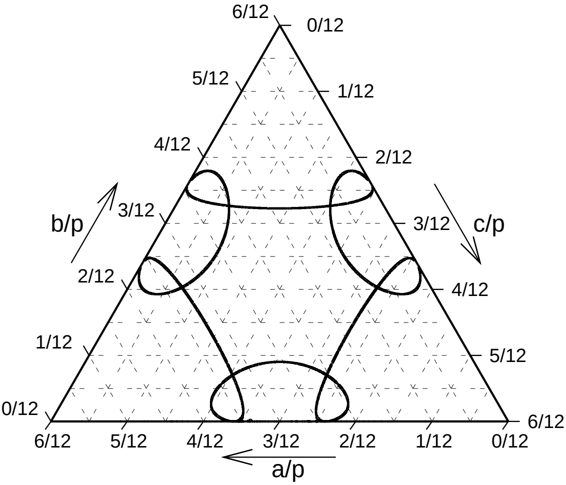


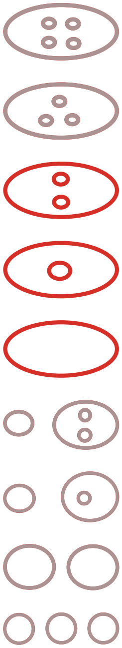
Perimeter cuts gallery: p=2.40



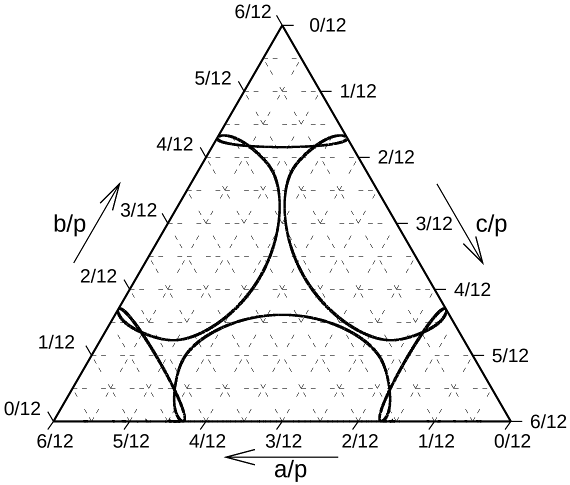


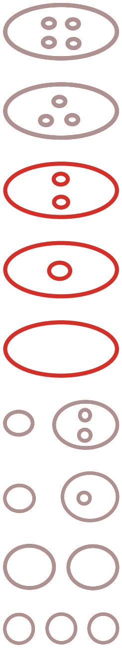
Perimeter cuts gallery: p=2.70



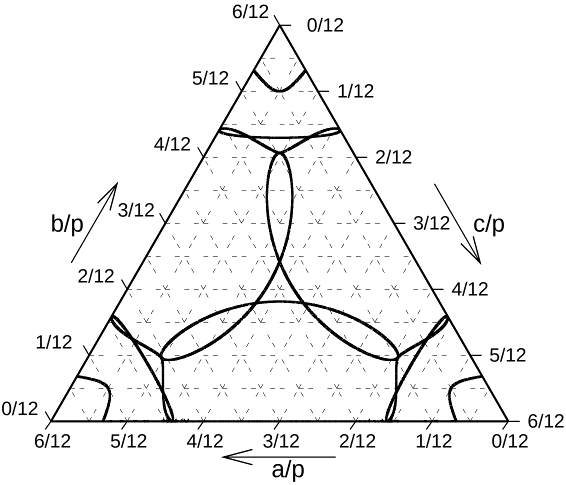


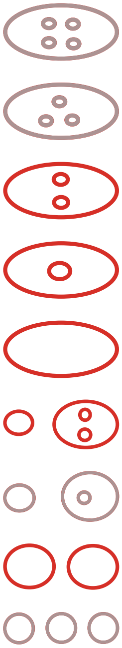
Perimeter cuts gallery: p=3.90



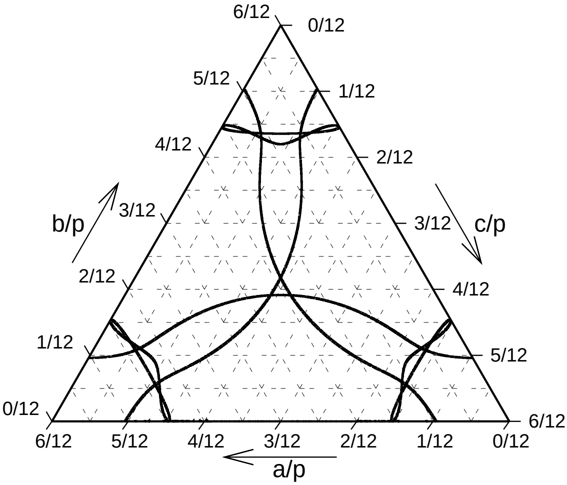


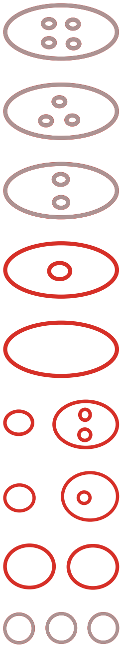
Perimeter cuts gallery: p=4.20



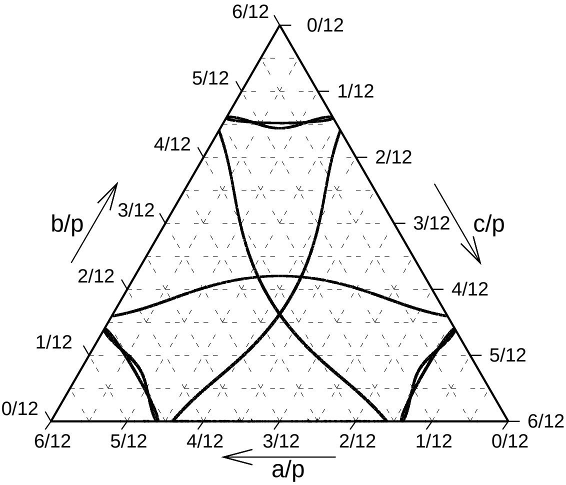


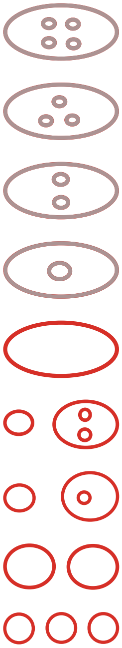
Perimeter cuts gallery: p=4.35



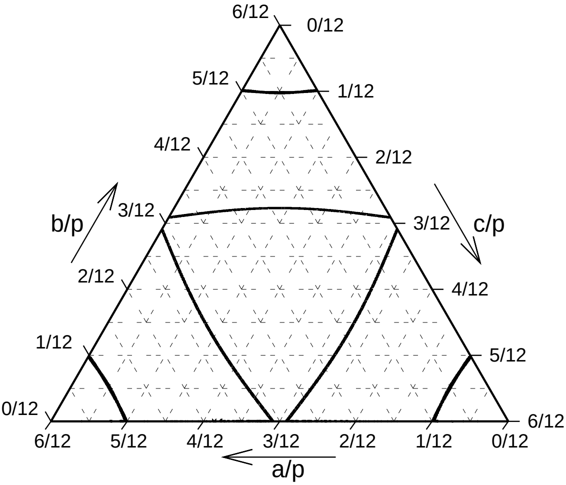


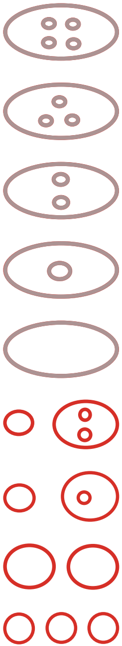
Perimeter cuts gallery: p=4.80





Perimeter cuts gallery: p=6.90

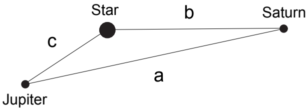


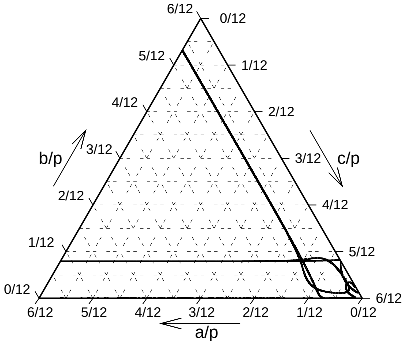


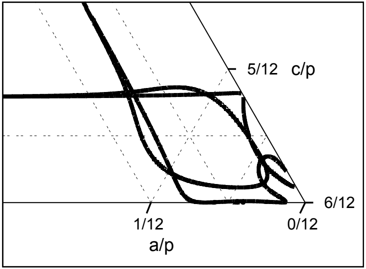
OGLE-2006-BLG-109L analog: general position

Sequence of ternary plots for different values of circumference.

Example: $p = 2.13$







OGLE-2006-BLG-109L analog: coplanar orbits