Difference-Imaging of Undersampled Data

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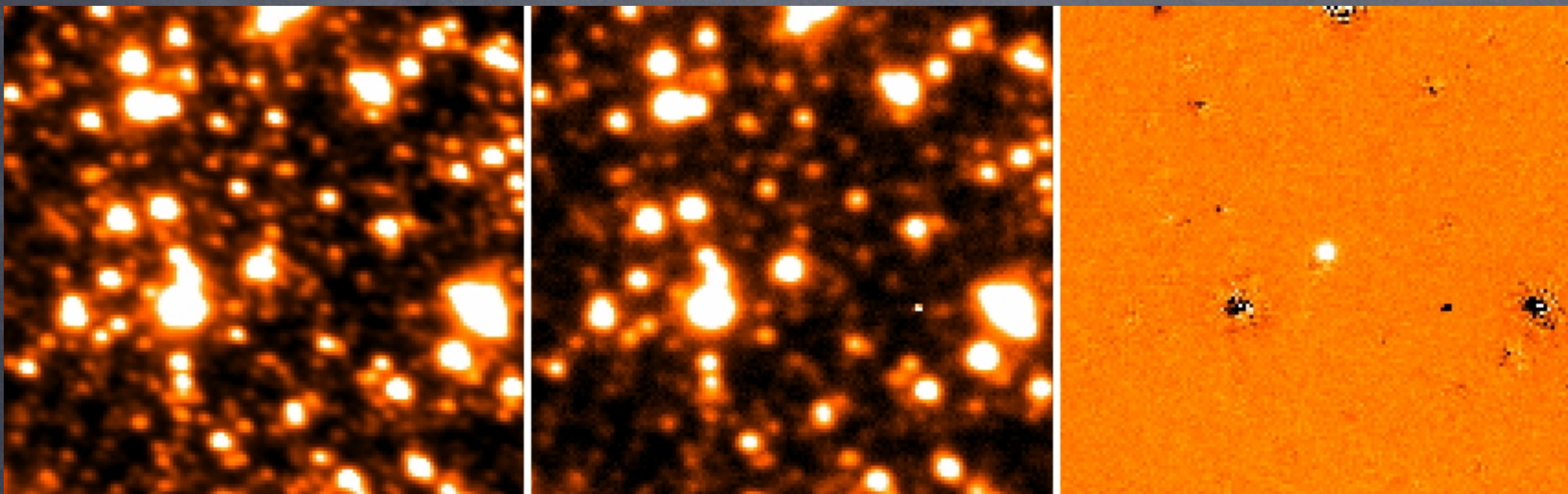
Difference-Imaging

If we have a reference image, R, and a series of target images, T^{α}, then we define the difference image

$$D^{\alpha} \equiv R \otimes K^{\alpha} - T^{\alpha}$$

K^{α} is a convolution kernel computed to minimize

$$\chi^2 = \sum_{ij} \left(\frac{D_{ij}}{\sigma_{ij}} \right)^2$$



R T D

Convolution Kernel

Analytic - e.g. sum of a few fixed width gaussians multiplied by polynomials (Alard 2000). (Almost) complete image registration required.

Numerical - grid of pixels (Bramich 2008). Registration by integer-pixel shifts OK. Can cope with weird PSFs.

In either case the kernel can be allowed to vary smoothly across the image.

Reference image

Generally we use either the single image with the best seeing, or use a (deconvolved) stack of good-seeing images.

Usually nature contrives to give us the best seeing when the sky background is the highest.

We can use an iterative approach to construct a reference image using the information present in all the images.

Consider the difference equation

$$D^{\alpha} \equiv R \otimes K^{\alpha} - T^{\alpha}$$

If we know K^{α}, we can compute the reference image R that minimizes

$$\chi^2 = \sum_{\alpha} \sum_{ij} \left(\frac{\left(R \otimes K^{\alpha}\right)_{ij} - T_{ij}^{\alpha}}{\sigma_{ij}^{\alpha}} \right)^2$$

After some algebra, the solution can be expressed as a matrix equation

$$A.\underline{r} = \underline{b}$$

$\underline{r}$ is a vector of image pixels in R

A is a matrix with elements

$$A_{(ij)(i'j')} = \sum_{\alpha} \sum_{pq} \frac{1}{\sigma^2_{\alpha pq}} K^{\alpha}_{(i-p)(j-q)} K^{\alpha}_{(i'-p)(j'-q)}$$

$\underline{b}$ is a vector with elements

$$b_{(ij)} = \sum_{\alpha} \sum_{pq} \frac{1}{\sigma^2_{\alpha pq}} T^{\alpha}_{pq} K^{\alpha}_{(i'-p)(j'-q)}$$

Matrix A is very large (dimension = number of image pixels) but is sparse and band-diagonal.

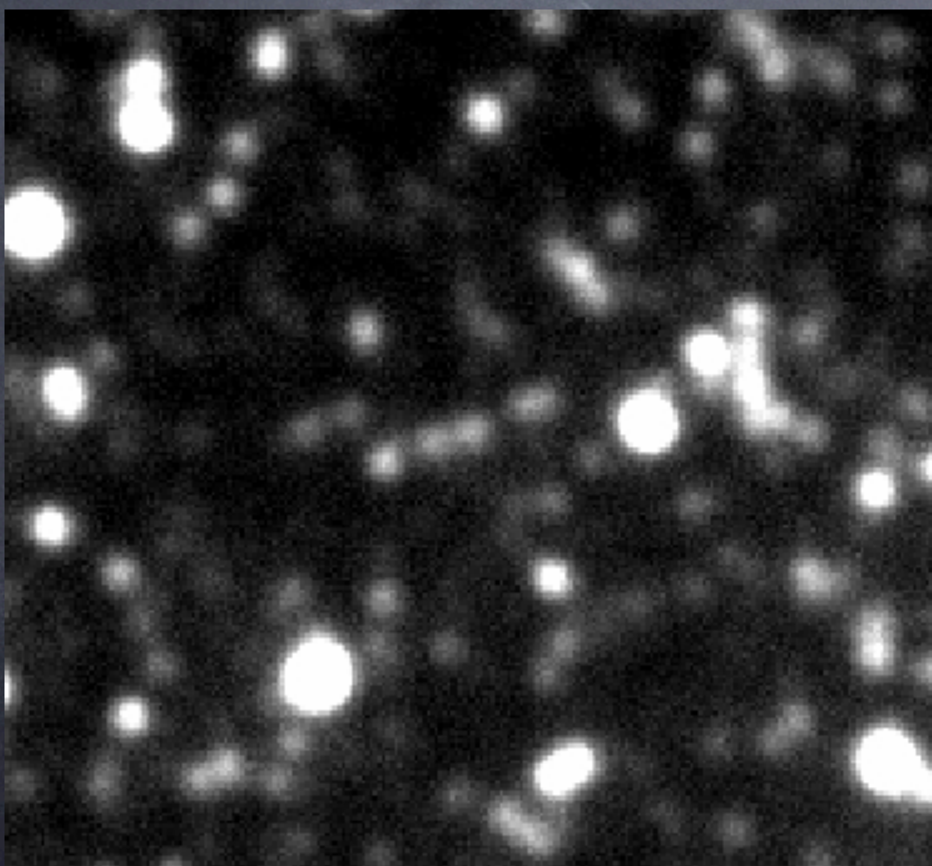
The computation is tractable with sparse-matrix methods.

Practical Procedure

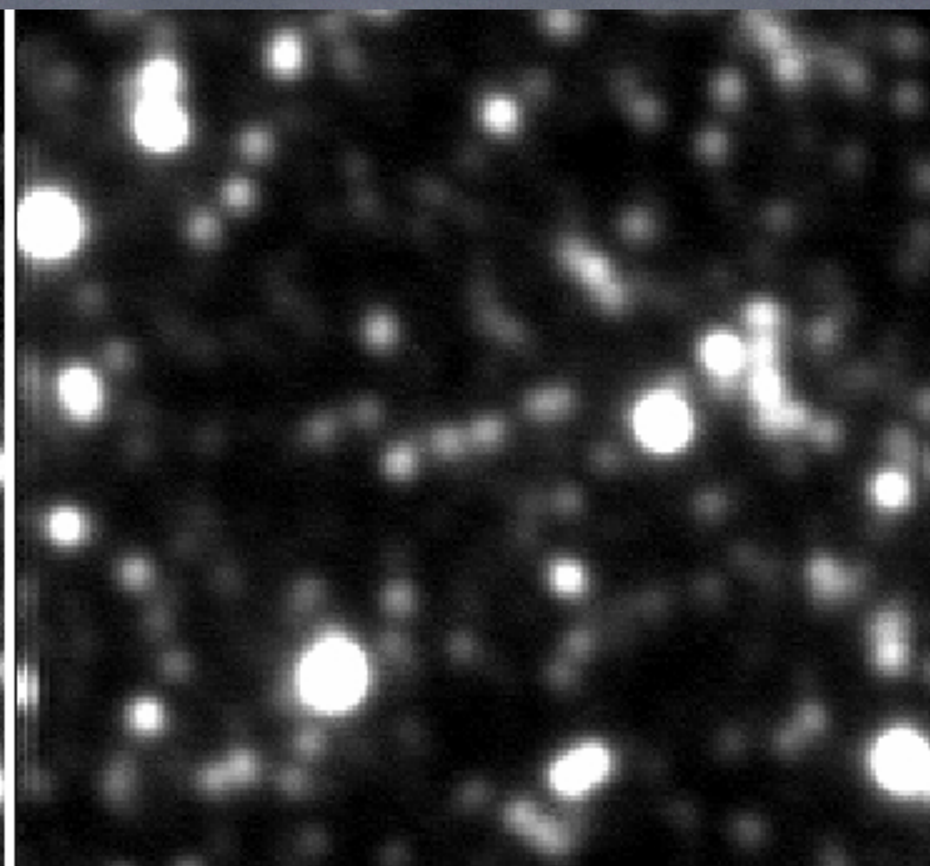
1. Adopt the best-seeing image as the initial reference.
2. Compute K^{α}
- Compute D.
- Compute a new R.
- Repeat steps 2-4.

UTas images for MOA 2011-BLG-262

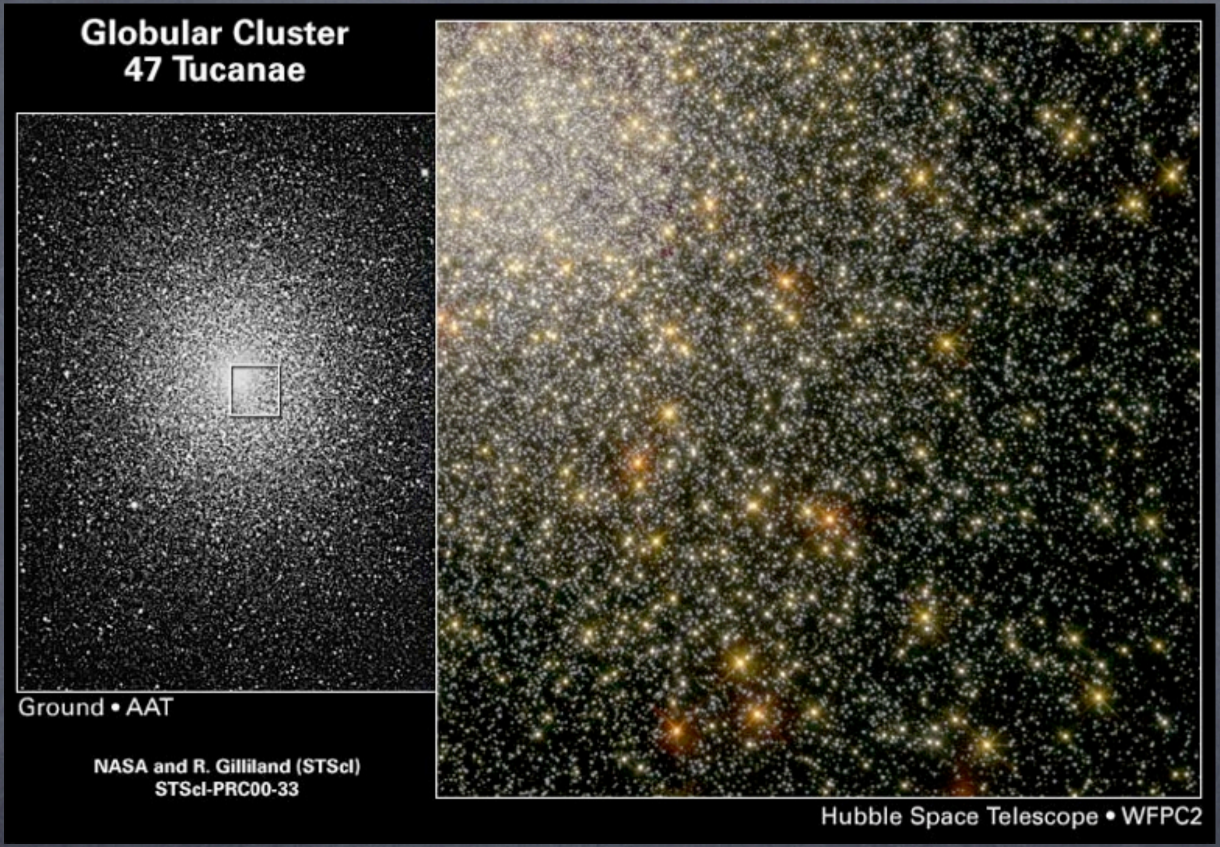
original reference



after 6 iterations



Undersampled Difference Imaging



Pioneering work by Ron Gilliland

Gilliland et al. (2000), Albrow et al. (2001), Sahu et al. (2006)

Undersampled Images

Cannot interpolate between pixels because there is insufficient information present to reconstruct a stellar PSF.

Each star in an image has a different effective PSF that depends on its subpixel location.

The convolution kernel required to map one image to another is different for every star.

Undersampled Images

The consequences of this are that

1. You cannot fully register a stack of images. Only integer-pixel shifts are allowed.

2. No reference image with the same sampling as the stack of target images will work effectively.

Instead, we need to construct an oversampled effective reference, which can then be shifted and evaluated at the undersampled pixel locations.

We can can use a similar procedure as before to construct an oversampled reference image using the information present in a series of images with subpixel XY dithers.

If we know K^{α}, we can compute the reference image R that minimizes

$$\chi^2 = \sum_{\alpha} \sum_{ij} \left(\frac{\left[R \otimes K^{\alpha} \right]_{ij} - T_{ij}^{\alpha}}{\sigma_{ij}^{\alpha}} \right)^2$$

where $[\ ]_{ij}$ means an oversampled quantity evaluated at undersampled location ij.

In this case, we have no initial reference image, but, by cross-correlation, we have initial estimates for the XY subpixel dithers. We can construct kernels that act purely as shift operators, i.e. of the form

(1-dx).dy	dx.dy
(1-dx).(1-dy)	dx.(1-dy)

From these shift-kernels, we can construct our oversampled reference image.

The XY offsets can be iteratively refined by expanding

$$R \rightarrow R_0 + \delta x \frac{\partial R}{\partial x} + \delta y \frac{\partial R}{\partial y}$$

Inserting this expression in the difference-imaging equation, we can solve for $(\delta x, \delta y)$ that minimize χ^2.

If we define the variance-weighted dot product and norm for images

$$\left\langle F \middle| G \right\rangle \equiv \sum_{ij} \frac{F_{ij} G_{ij}}{\sigma_{ij}^2} \qquad \left| F \right|^2 \equiv \left\langle F \middle| F \right\rangle$$

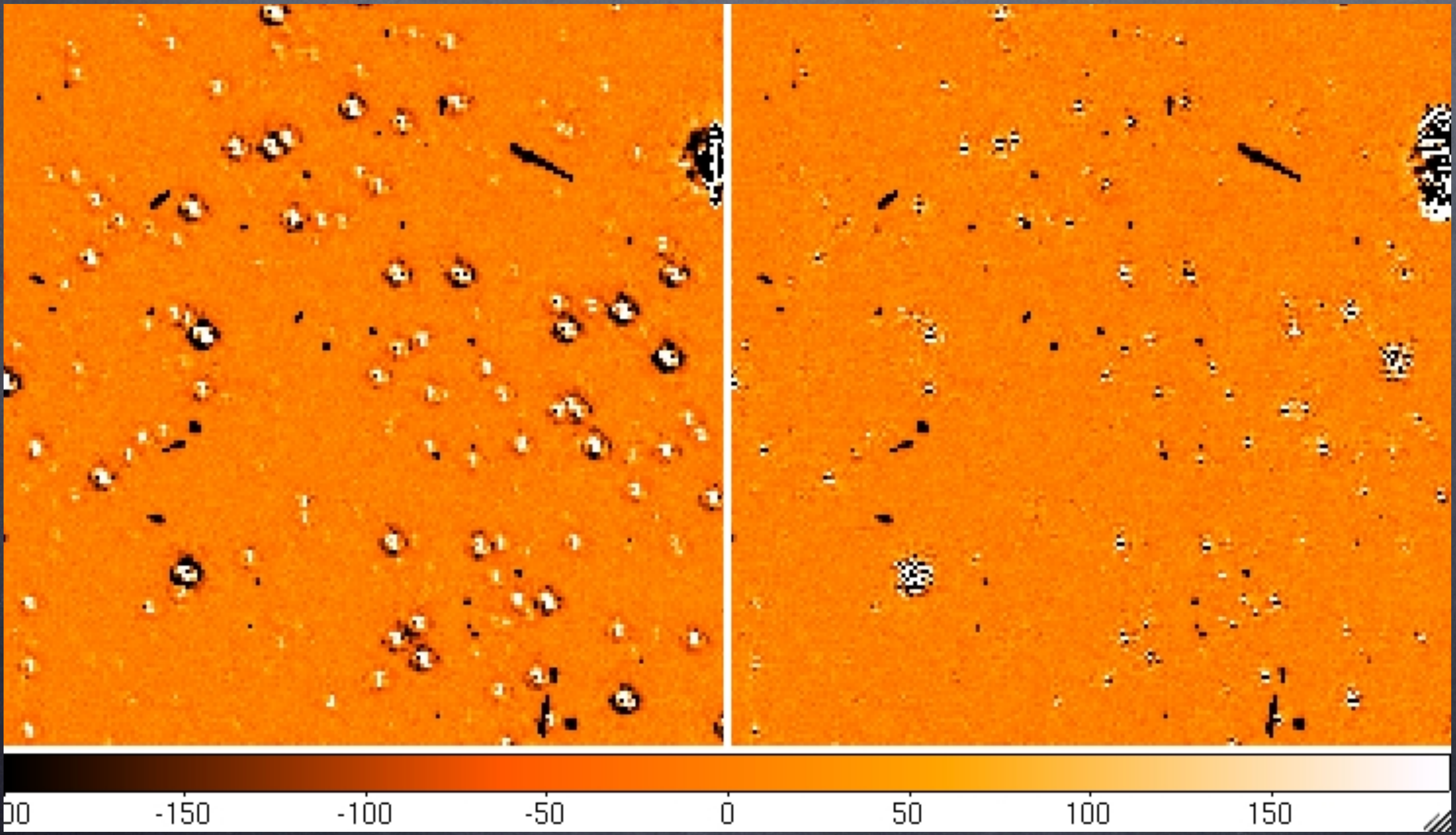
then

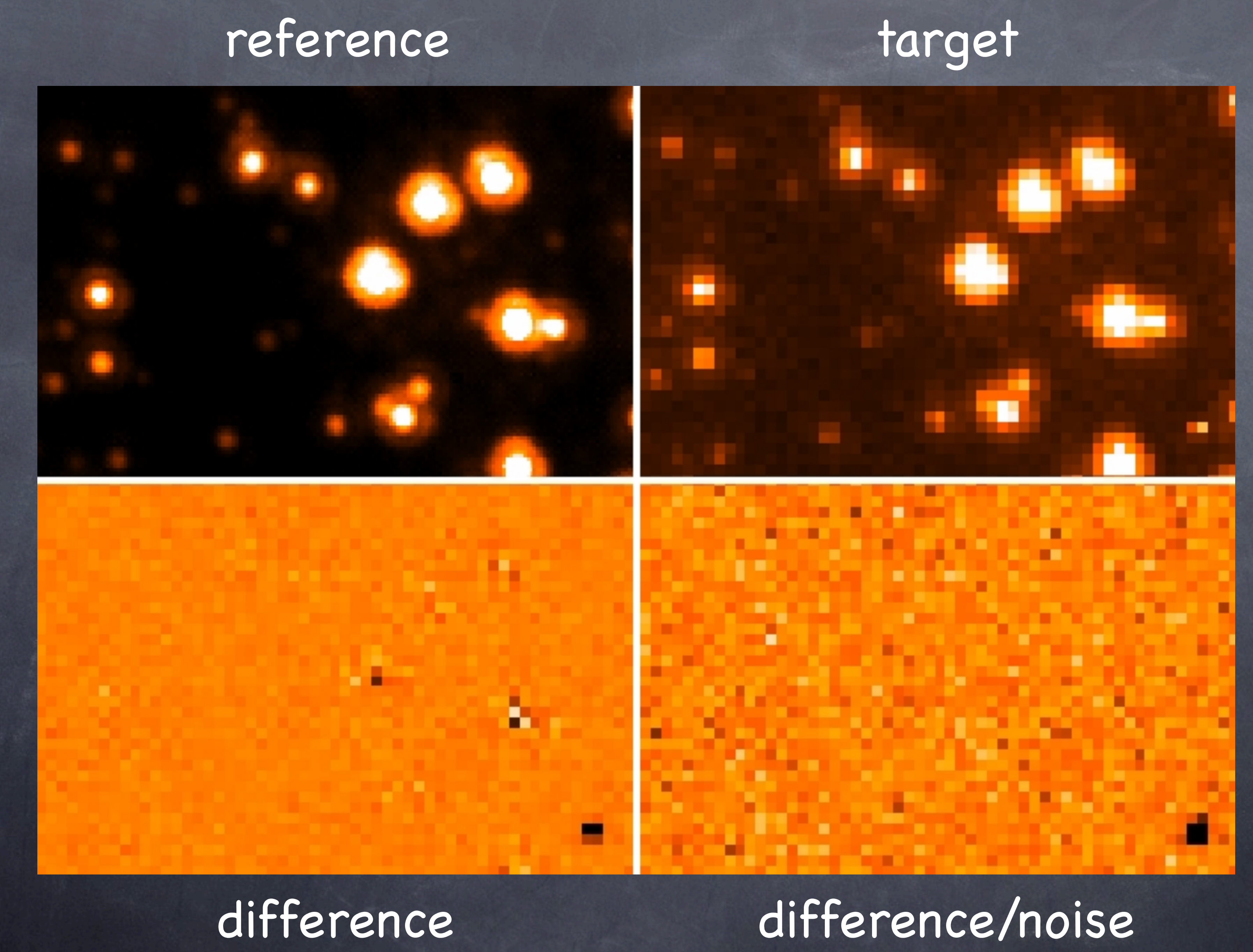
$$\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = H^{-1} . \begin{pmatrix} -\left\langle D_0 \middle| \frac{\partial R}{\partial x} \otimes K \right\rangle \\ -\left\langle D_0 \middle| \frac{\partial R}{\partial y} \otimes K \right\rangle \end{pmatrix}$$

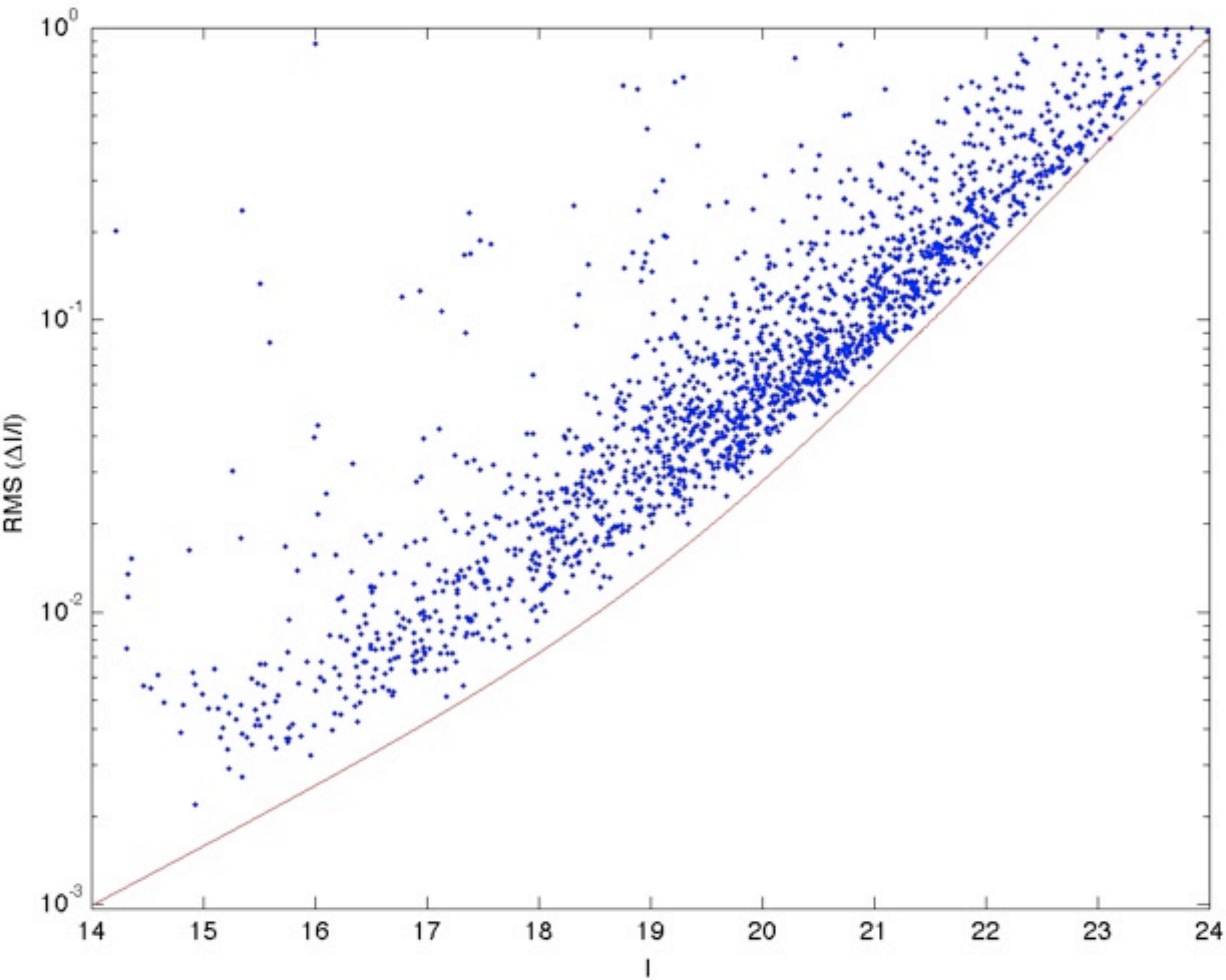
where

$$H = \begin{pmatrix} \left| \frac{\partial R}{\partial x} \otimes K \right|^2 & \left\langle \frac{\partial R}{\partial y} \otimes K \middle| \frac{\partial R}{\partial y} \otimes K \right\rangle \\ \left\langle \frac{\partial R}{\partial y} \otimes K \middle| \frac{\partial R}{\partial y} \otimes K \right\rangle & \left| \frac{\partial R}{\partial y} \otimes K \right|^2 \end{pmatrix}$$

For HST, need to add final convolution to account for “breathing”.

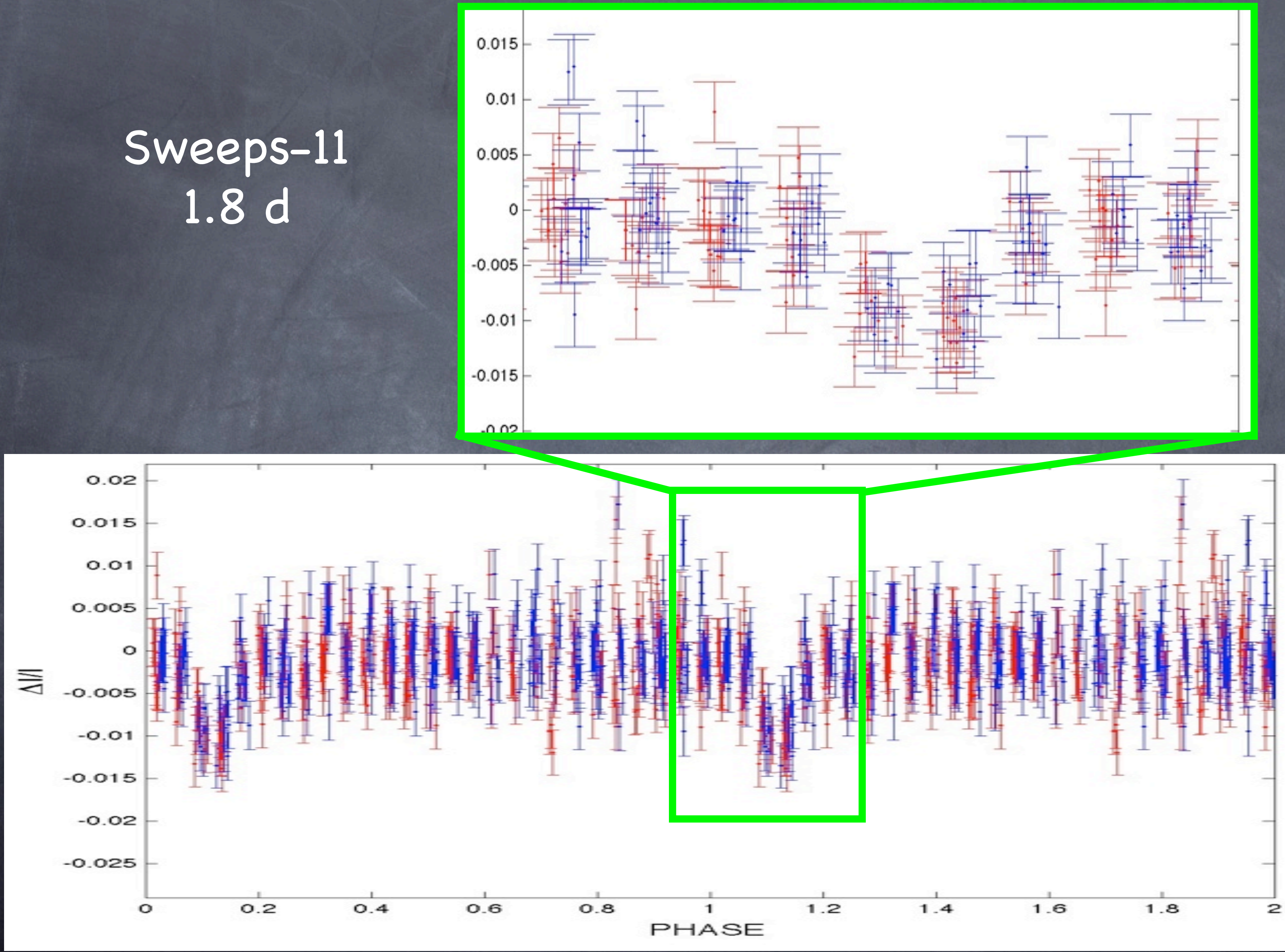



reference
target
difference
difference/noise

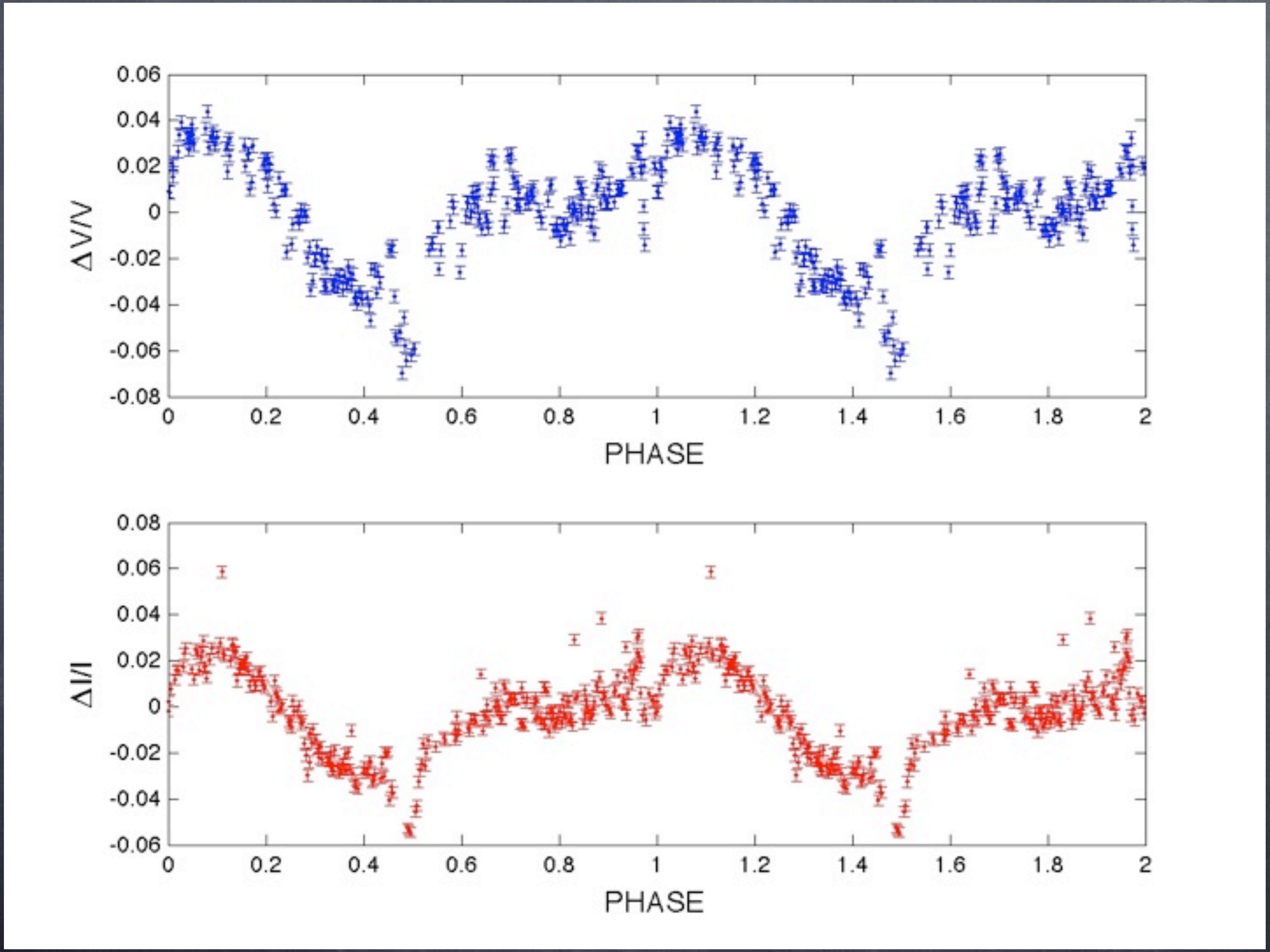


RMS (ΔI/I)
I

Sweeps-11
1.8 d



P = 1.85 d



0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
ΔV/V
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
PHASE
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
ΔI/I
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
PHASE

WFIRST

Pixel size 0.18”

Diffraction limit at 1.5 micron (centre of W149) = 0.30” (a little below critical sampling) -> need to dither

ExP plan is for no explicit dithering

Coarse-pointing accuracy <= 3’ RMS -> OK

FGS pointing to < 0.025” is too good. If FGS is used for return visits, need to have programmed dithers.